Administrative Normal Forms and Focusing for Lambda Calculi

David Binder, Thomas Piecha, Colloquium Logicum 2022, Konstanz

A bird's-eye view

Some context in lieu of motivation

- Compilation of functional programming languages and proof assistants based on **type theory**.
- **Efficient** implementation of logic on a computer.
- Use of **abstract machines** as an intermediary between substitution based evaluation (e.g beta-reduction) and low-level machines (e.g. x86-Assembly).

A Tale of Two Calculi

Natural Deduction and Sequent Calculus

- Natural deduction (ND) and Sequent Calculus (SC) developed in the same paper by G. Gentzen (1935).
- Curry and Howard discover the relation between combinatory logic and Hilbert calculus, lambda calculus and ND, but no similar system for classical SC is found.
- Griffin (1989) discovers that the type of control operator call/cc is Peirce's law.
- Development of $\lambda\mu$-calculus by Parigot (1992), and $\lambda\mu\tilde{\mu}$-calculus by Curien and Herbelin (2000).
- Since then: Lot of work on $\lambda\mu\tilde{\mu}$-calculus; Ariola, Downen, Munch-Maccagnoni, Zeilberger, Wadler...

Sequent Calculus as a Compiler Intermediate Language

Paul Downen Luke Maurer
Zena M. Ariola
University of Oregon, USA
{pdownen,maurerl,ariola}@cs.uoregon.edu

Simon Peyton Jones
Microsoft Research Cambridge, UK
simonpj@microsoft.com

Relating ND and SC

Two sides of the same coin

- Two communities: Logicians/proof theorists and PLT people.
- **Focusing** was introduced by Andreoli (1992) for proof search in linear logic.
- The **A-normal form** was introduced by Sabry and Felleisen (1992) as an optimization of CPS based compilers.
- We show how to relate the two.

Example

Example (ND)

$$\pi_2(\pi_1(1, 4), 3)$$

Example (ND)

$$\pi_2(\pi_1(1, 4), 3)$$

Tupling!

Example (ND)

$$\pi_2(\pi_1(1, 4), 3)$$

Projections!

Example (ND)

$$\pi_2(\pi_1(1, 4), 3)$$

Numbers!

Example (ND)

$$\pi_2(\pi_1(1, 4), 3)$$

Should evaluate in one step to $\pi_2(1,3)$ (CBV!)

Example (ND)

Redex

Evaluation context $\pi_2(\pi_1(1, 4), 3)$

Very inefficient! We want to get rid of search for next redex.

Example (ND)

$$\pi_2(\pi_1(1,4),3)$$

Compile to A-normal form:

$$\textbf{let } x = \pi_1(1,4) \textbf{ in } (\textbf{let } y = \pi_2(x,3) \textbf{ in } y)$$

Enter the sequent calculus

Example (SC)

The same example in sequent calculus

ND: $$\pi_2(\pi_1(1,4),3)$$

SC: $$\mu\alpha.\langle(\mu\beta.\langle(1,4) \mid \pi_1\ \beta\rangle,3) \mid \pi_2\ \alpha\rangle.$$

Example (SC)

The same example in sequent calculus

ND: $$\pi_2(\pi_1(1,4),3)$$

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Example (SC)

The same example in sequent calculus

ND: $$\pi_2(\pi_1(1,4),3)$$

SC: $$\mu\alpha.\langle(\mu\beta.\langle(1,4) \mid \pi_1\ \beta\rangle,3) \mid \pi_2\ \alpha\rangle.$$

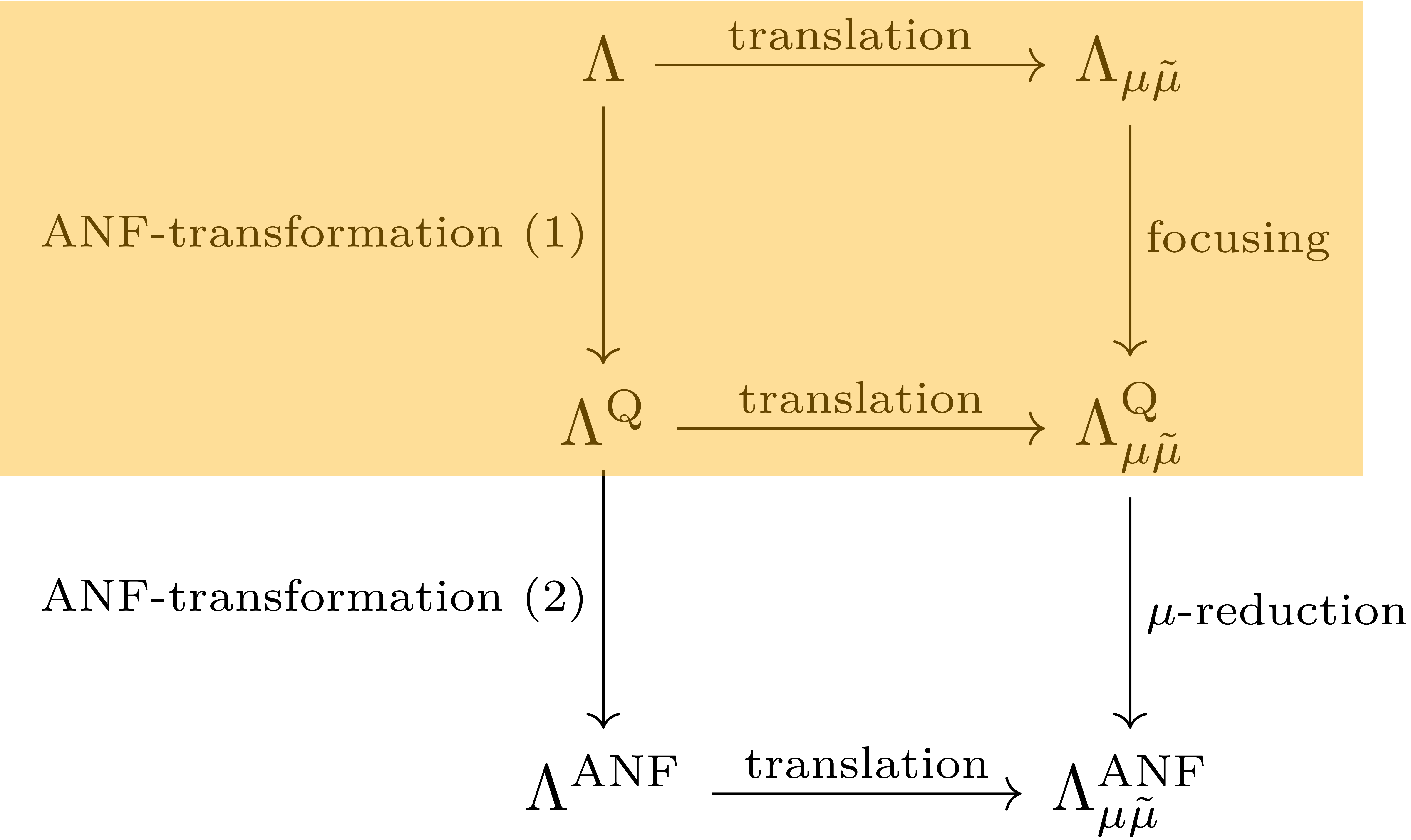
Example (SC)

The focused normal form

$$\mu\alpha.\langle\mu\beta.\langle(1,4) \mid \pi_1\ \beta\rangle \mid \tilde{\mu}x.\langle(x,3) \mid \pi_2\ \alpha\rangle\rangle.$$

$$\textbf{let}\ x = \pi_1(1,4)\ \textbf{in}\ (\textbf{let}\ y = \pi_2(x,3)\ \textbf{in}\ y)$$

Overview

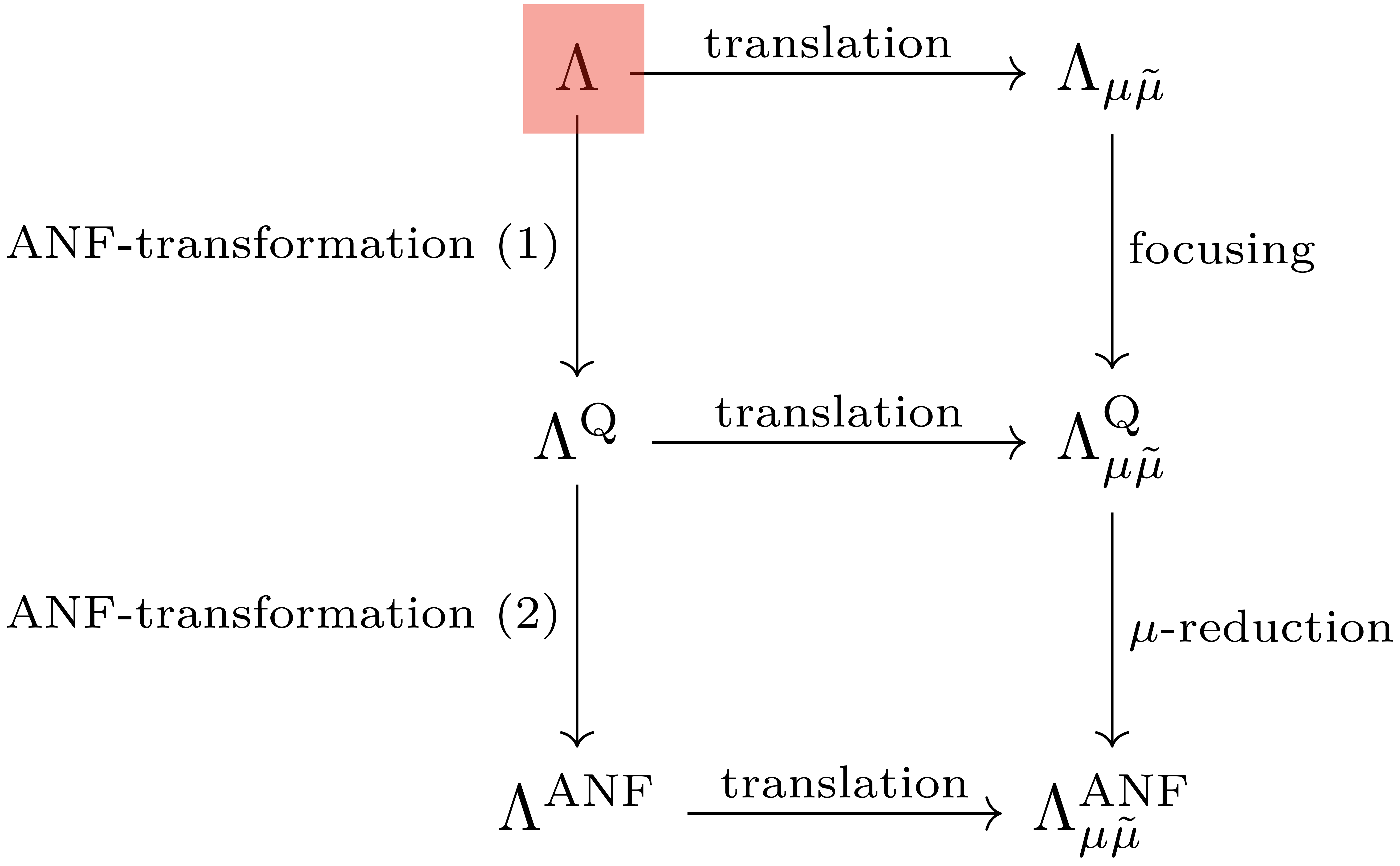


Λ
translation
$\Lambda_{\mu\tilde{\mu}}$
ANF-transformation (1)
focusing
Λ^{Q}
translation
$\Lambda^{Q}_{\mu\tilde{\mu}}$
ANF-transformation (2)
μ-reduction
Λ^{ANF}
translation
$\Lambda^{ANF}_{\mu\tilde{\mu}}$

The Lambda Calculus

Lambda Calculus

Overview



Lambda Calculus

Types

$$\tau ::= X \mid \tau \to \tau \mid \tau \wedge \tau.$$

Lambda Calculus

Terms

$$\textit{Terms:} \quad e ::= x \mid \lambda x.e \mid e\ e \mid (e, e) \mid \pi_1\ e \mid \pi_2\ e \mid \textbf{let}\ x = e\ \textbf{in}\ e.$$

$$\textit{Values:} \quad v ::= \lambda x.e \mid (v, v) \mid x.$$

Lambda Calculus

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Lambda Calculus

Typing rules

$$\frac{}{\Gamma, x : \tau \vdash x : \tau}\ \textsc{Var}$$

$$\frac{\Gamma \vdash e_1 : \sigma \qquad \Gamma, x : \sigma \vdash e_2 : \tau}{\Gamma \vdash \textbf{let}\ x = e_1\ \textbf{in}\ e_2 : \tau}\ \textsc{Let}$$

$$\frac{\Gamma, x : \sigma \vdash e : \tau}{\Gamma \vdash \lambda x.e : \sigma \to \tau}\ \textsc{Abs}$$

$$\frac{\Gamma \vdash e_1 : \sigma \to \tau \qquad \Gamma \vdash e_2 : \sigma}{\Gamma \vdash e_1\ e_2 : \tau}\ \textsc{App}$$

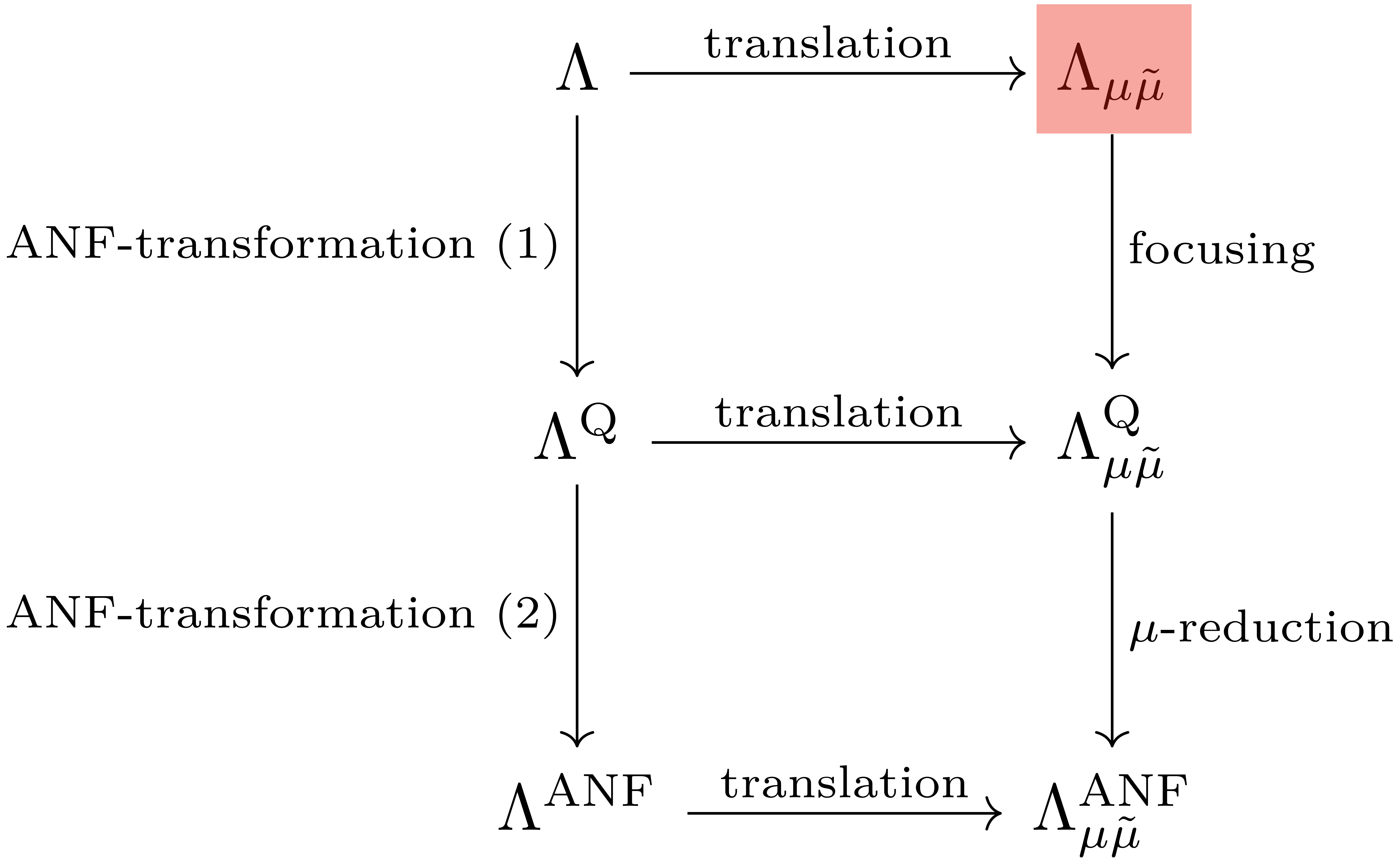
$$\frac{\Gamma \vdash e_1 : \sigma \qquad \Gamma \vdash e_2 : \tau}{\Gamma \vdash (e_1, e_2) : \sigma \wedge \tau}\ \textsc{Pair}$$

$$\frac{\Gamma \vdash e : \tau_1 \wedge \tau_2}{\Gamma \vdash \pi_i\ e : \tau_i}\ \textsc{Proj}$$

The Sequent Calculus

Sequent Calculus

Overview



Sequent Calculus

Terms

$$\textit{Terms: } t ::= x \mid \lambda x.t \mid (t, t) \mid \mu\alpha.c.$$

$$\textit{Coterms: } s ::= \alpha \mid t \cdot s \mid \pi_1\, s \mid \pi_2\, s \mid \tilde{\mu}x.c.$$

$$\textit{Commands: } c ::= \langle t \mid s \rangle.$$

$$\textit{Values: } \quad w ::= \lambda x.t \mid (w, w) \mid x.$$

Judgement forms

1. $\Gamma \vdash [\varphi], \Delta$

   "If all $\gamma \in \Gamma$ are true and all $\delta \in \Delta$ are false, then φ is true."

2. $\Gamma, [\varphi] \vdash \Delta$

   "If all $\gamma \in \Gamma$ are true and all $\delta \in \Delta$ are false, then φ is false."

3. $\Gamma \vdash \Delta$

   "The assumption that all $\gamma \in \Gamma$ are true and all $\delta \in \Delta$ are false is contradictory."

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Sequent Calculus

Terms

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$$\textit{Values:} \quad v ::= \lambda x.e \mid (v, v) \mid x.$$

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Sequent Calculus

Typing rules

Term typing

$$\frac{}{\Gamma, x : \tau \vdash x : \tau \mid \Delta} \textsc{Var}_x$$

$$\frac{\Gamma, x : \sigma \vdash t : \tau \mid \Delta}{\Gamma \vdash \lambda x.t : \sigma \to \tau \mid \Delta} \textsc{Abs}$$

$$\frac{\Gamma \vdash t_1 : \tau_1 \mid \Delta \qquad \Gamma \vdash t_2 : \tau_2 \mid \Delta}{\Gamma \vdash (t_1, t_2) : \tau_1 \wedge \tau_2 \mid \Delta} \textsc{Pair}$$

$$\frac{c : (\Gamma \vdash \alpha : \tau, \Delta)}{\Gamma \vdash \mu\alpha.c : \tau \mid \Delta} \textsc{Mu}$$

Coterm typing

$$\frac{}{\Gamma \mid \alpha : \tau \vdash \alpha : \tau, \Delta} \textsc{Var}_\alpha$$

$$\frac{\Gamma \vdash t : \tau \mid \Delta \qquad \Gamma \mid s : \sigma \vdash \Delta}{\Gamma \mid t \cdot s : \tau \to \sigma \vdash \Delta} \textsc{App}$$

$$\frac{\Gamma \mid s : \tau_i \vdash \Delta}{\Gamma \mid \pi_1\ s : \tau_1 \wedge \tau_2 \vdash \Delta} \textsc{Proj}$$

$$\frac{c : (\Gamma, x : \tau \vdash \Delta)}{\Gamma \mid \tilde{\mu}x.c \vdash \Delta} \textsc{Mu}_\sim$$

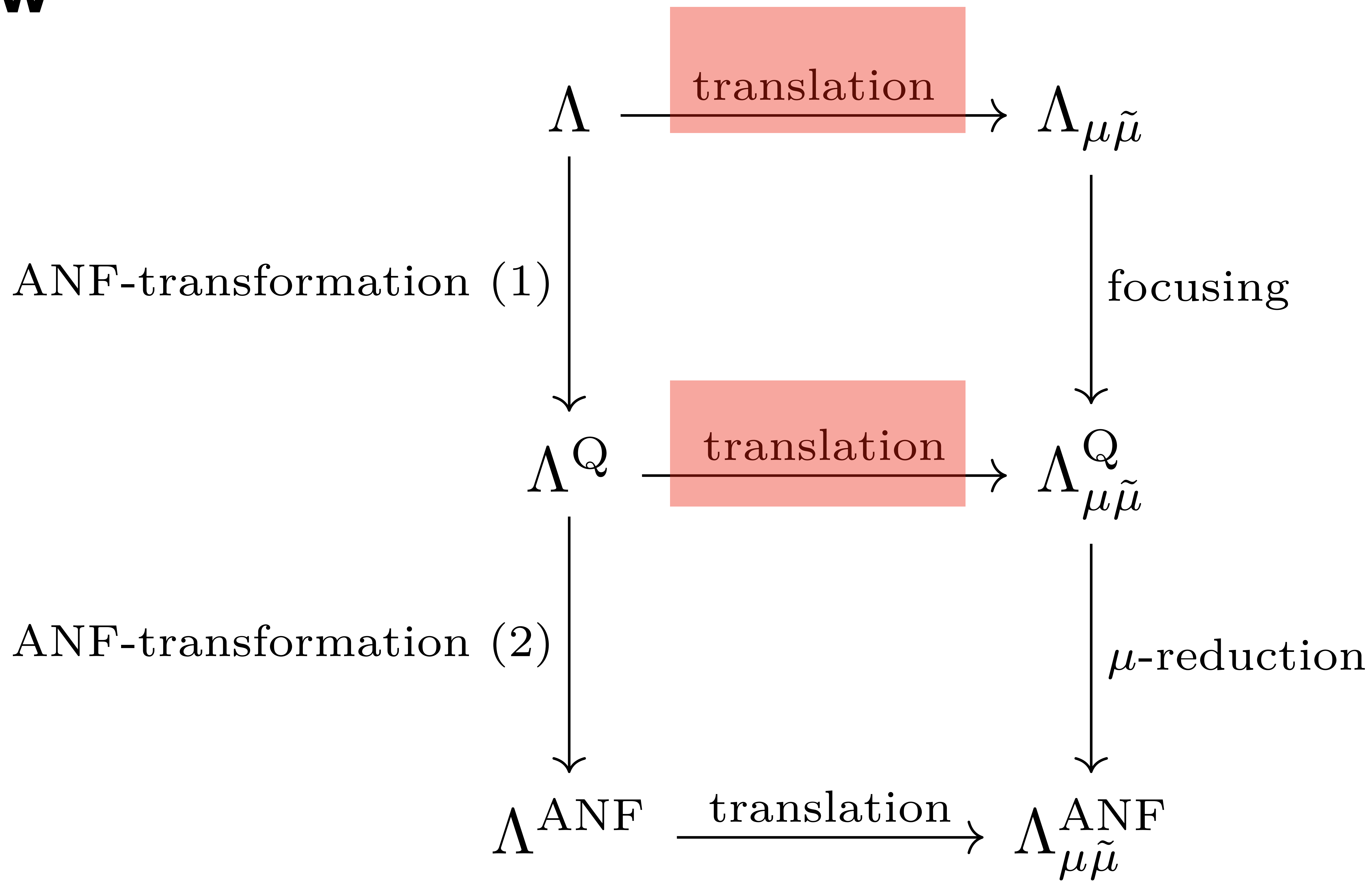
Command typing

$$\frac{\Gamma \vdash t : \tau \mid \Delta \qquad \Gamma \mid s : \tau \vdash \Delta}{\langle t \mid s \rangle : (\Gamma \vdash \Delta)} \textsc{Cut}$$

Translating from ND to SC

Sequent Calculus

Overview



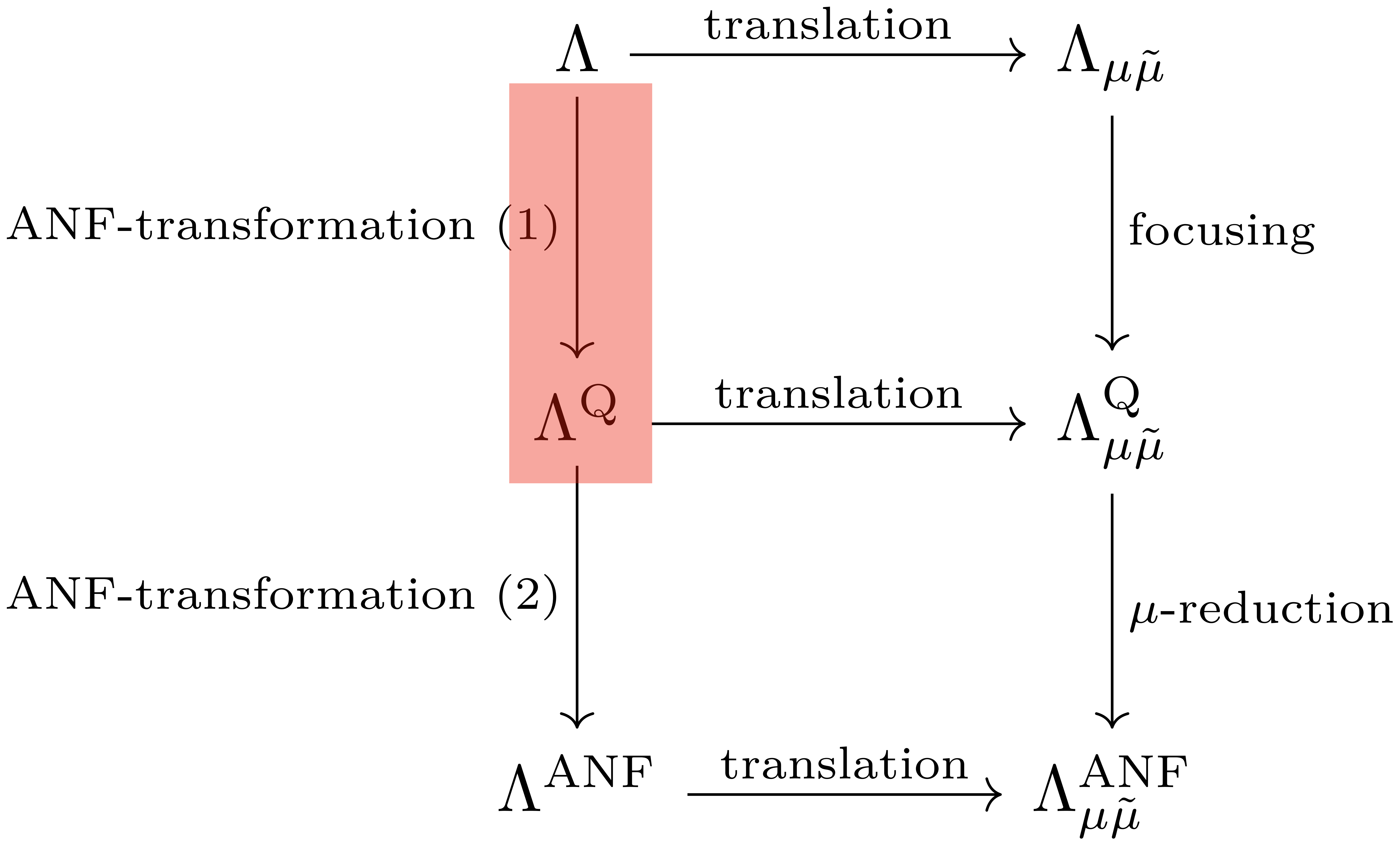
Translation

$$
\begin{aligned}
[\![x]\!] &:\triangleq x \\
[\![\lambda x.e]\!] &:\triangleq \lambda x.[\![e]\!] \\
[\![(e_1, e_2)]\!] &:\triangleq ([\![e_1]\!], [\![e_2]\!]) \\
[\![e_1\ e_2]\!] &:\triangleq \mu\alpha.\langle [\![e_1]\!] \mid [\![e_2]\!] \cdot \alpha \rangle \\
[\![\pi_i\ e]\!] &:\triangleq \mu\alpha.\langle [\![e]\!] \mid \pi_i\ \alpha \rangle \\
[\![\mathbf{let}\ x = e_1\ \mathbf{in}\ e_2]\!] &:\triangleq \mu\alpha.\langle [\![e_1]\!] \mid \tilde{\mu}x.\langle [\![e_2]\!] \mid \alpha \rangle \rangle.
\end{aligned}
$$

The Q Normal Form for ND

Sequent Calculus

Overview



The Q-normal form for ND

Before

$$\textit{Terms:} \quad e ::= x \mid \lambda x.e \mid e\ e \mid (e, e) \mid \pi_1\ e \mid \pi_2\ e \mid \mathbf{let}\ x = e\ \mathbf{in}\ e.$$

$$\textit{Values:} \quad v ::= \lambda x.e \mid (v, v) \mid x.$$

After

$$\textit{Values:} \quad v ::= \lambda x.e \mid (v, v) \mid x.$$

$$\textit{Terms:} \quad e ::= v \mid \mathbf{let}\ x = e\ \mathbf{in}\ e \mid e\ v \mid \pi_1\ e \mid \pi_2\ e.$$

The Q-normal form for ND

Before

$$\textit{Terms:} \quad e ::= x \mid \lambda x.e \mid e\ e \mid (e, e) \mid \pi_1\ e \mid \pi_2\ e \mid \textbf{let}\ x = e\ \textbf{in}\ e.$$

$$\textit{Values:} \quad v ::= \lambda x.e \mid (v, v) \mid x.$$

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$$\textit{Values:} \quad v ::= \lambda x.e \mid (v, v) \mid x.$$

$$\textit{Terms:} \quad e ::= v \mid \textbf{let}\ x = e\ \textbf{in}\ e \mid e\ v \mid \pi_1\ e \mid \pi_2\ e.$$

The Q-normal form for ND

Before

$$\textit{Terms:} \quad e ::= x \mid \lambda x.e \mid e\ e \mid (e, e) \mid \pi_1\ e \mid \pi_2\ e \mid \textbf{let}\ x = e\ \textbf{in}\ e.$$

$$\textit{Values:} \quad v ::= \lambda x.e \mid (v, v) \mid x.$$

After

$$\textit{Values:} \quad v ::= \lambda x.e \mid (v, v) \mid x.$$

$$\textit{Terms:} \quad e ::= v \mid \textbf{let}\ x = e\ \textbf{in}\ e \mid e\ v \mid \pi_1\ e \mid \pi_2\ e.$$

Translate into Q-normal form

$$\mathcal{A}(x) :\simeq x \qquad (\mathcal{A}_1)$$

$$\mathcal{A}(\lambda x.e) :\simeq \lambda x.\mathcal{A}(e) \qquad (\mathcal{A}_2)$$

$$\mathcal{A}(\textbf{let } x = e_1 \textbf{ in } e_2) :\simeq \textbf{let } x = \mathcal{A}(e_1) \textbf{ in } \mathcal{A}(e_2) \qquad (\mathcal{A}_3)$$

$$\mathcal{A}(\pi_i \ e) :\simeq \pi_i(\mathcal{A}(e)) \qquad (\mathcal{A}_4)$$

$$\mathcal{A}((v_1, v_2)) :\simeq (\mathcal{A}(v_1), \mathcal{A}(v_2)) \qquad (\mathcal{A}_5)$$

$$\mathcal{A}((v_1, e_2)) :\simeq \textbf{let } x = \mathcal{A}(e_2) \textbf{ in } (\mathcal{A}(v_1), x) \qquad (\mathcal{A}_6)$$

$$\mathcal{A}((e_1, v_2)) :\simeq \textbf{let } x = \mathcal{A}(e_1) \textbf{ in } (x, v_2) \qquad (\mathcal{A}_7)$$

$$\mathcal{A}((e_1, e_2)) :\simeq \textbf{let } x = \mathcal{A}(e_1) \textbf{ in } (\textbf{let } y = \mathcal{A}(e_2) \textbf{ in } (x, y)) \qquad (\mathcal{A}_8)$$

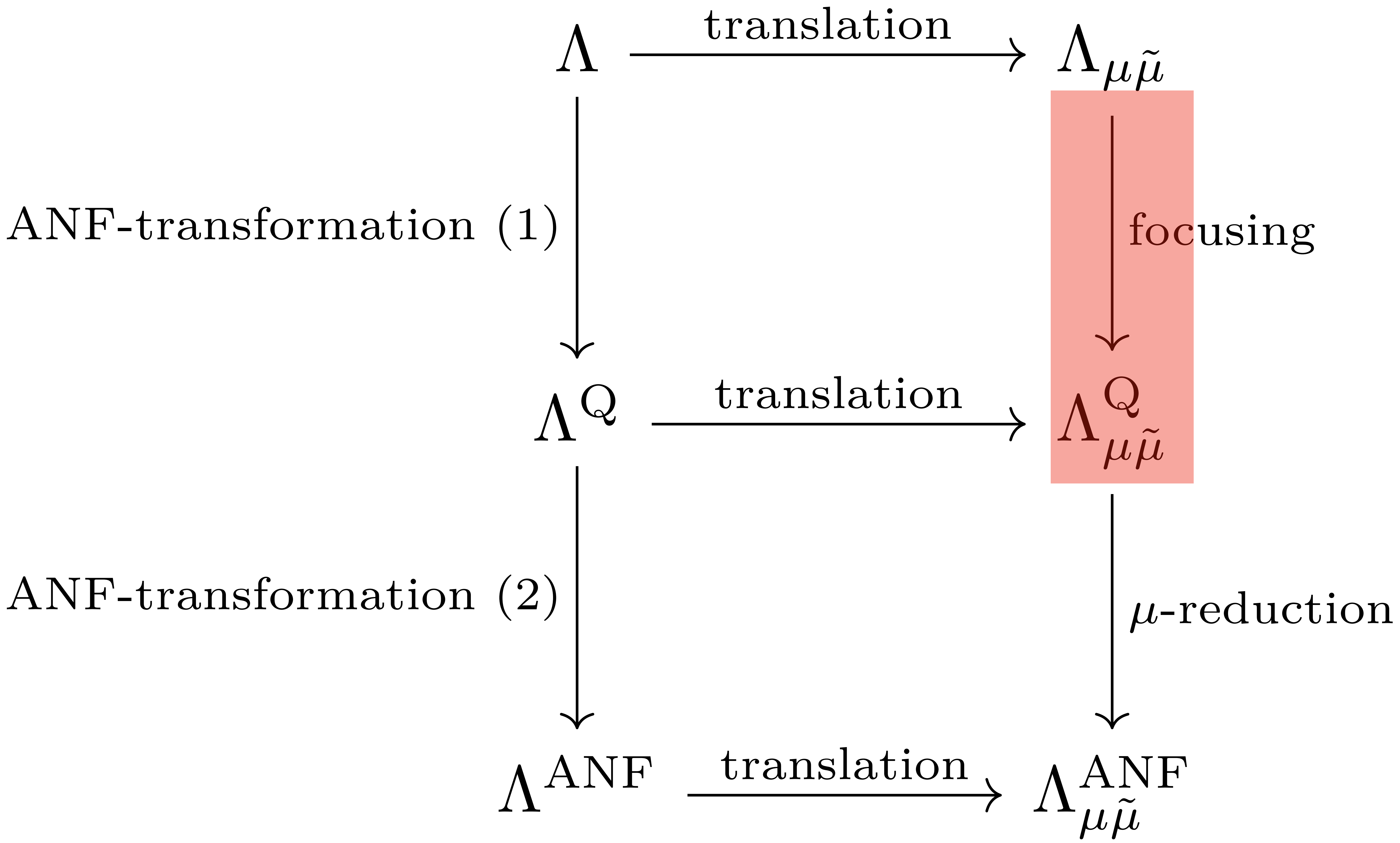
$$\mathcal{A}(e_1 \ v_2) :\simeq \mathcal{A}(e_1)\ \mathcal{A}(v_2) \qquad (\mathcal{A}_9)$$

$$\mathcal{A}(e_1 \ e_2) :\simeq \textbf{let } x = \mathcal{A}(e_2) \textbf{ in } \mathcal{A}(e_1)\ x \qquad (\mathcal{A}_{10})$$

The Q Normal Form for SC

Sequent Calculus

Overview



The Q-normal form for SC

Before

$$Terms:\ t ::= x \mid \lambda x.t \mid (t,t) \mid \mu\alpha.c.$$

$$Coterms:\ s ::= \alpha \mid t \cdot s \mid \pi_1\ s \mid \pi_2\ s \mid \tilde{\mu}x.c.$$

$$Commands:\ c ::= \langle t \mid s \rangle.$$

$$Values:\quad w ::= \lambda x.t \mid (w,w) \mid x.$$

After

$$Terms:\ t ::= w \mid \mu\alpha.c.$$

$$Coterms:\ s ::= \alpha \mid w \cdot s \mid \pi_1\ s \mid \pi_2\ s \mid \tilde{\mu}x.c.$$

$$Commands:\ c ::= \langle t \mid s \rangle.$$

$$Values:\quad w ::= \lambda x.t \mid (w,w) \mid x.$$

The Q-normal form for SC

Before

$$\textit{Terms: } t ::= x \mid \lambda x.t \mid (t,t) \mid \mu\alpha.c.$$

$$\textit{Coterms: } s ::= \alpha \mid t \cdot s \mid \pi_1\ s \mid \pi_2\ s \mid \tilde{\mu}x.c.$$

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After

$$\textit{Terms: } t ::= w \mid \mu\alpha.c.$$

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The Q-normal form for SC

Before

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$$Values:\quad w ::= \lambda x.t \mid (w,w) \mid x.$$

Translate into Q-normal form

Terms

$$\mathcal{F}(x) :\simeq x \qquad (\mathcal{F}_1)$$

$$\mathcal{F}(\mu\alpha.c) :\simeq \mu\alpha.\mathcal{F}(c) \qquad (\mathcal{F}_2)$$

$$\mathcal{F}(\lambda x.e) :\simeq \lambda x.\mathcal{F}(e) \qquad (\mathcal{F}_3)$$

$$\mathcal{F}((w_1, w_2)) :\simeq (\mathcal{F}(w_1), \mathcal{F}(w_2)) \qquad (\mathcal{F}_4)$$

$$\mathcal{F}((w_1, t_2)) :\simeq \mu\alpha.\langle \mathcal{F}(t_2) \mid \tilde{\mu}x.\langle (\mathcal{F}(w_1), x) \mid \alpha\rangle\rangle \qquad (\mathcal{F}_5)$$

$$\mathcal{F}((t_1, w_2)) :\simeq \mu\alpha.\langle \mathcal{F}(t_1) \mid \tilde{\mu}x.\langle (x, \mathcal{F}(w_2)) \mid \alpha\rangle\rangle \qquad (\mathcal{F}_6)$$

$$\mathcal{F}((t_1, t_2)) :\simeq \mu\alpha.\langle \mathcal{F}(t_1) \mid \tilde{\mu}x.\langle \mu\beta.\langle \mathcal{F}(t_2) \mid \tilde{\mu}y.\langle (x, y) \mid \beta\rangle\rangle \mid \alpha\rangle\rangle \qquad (\mathcal{F}_7)$$

Coterms

$$\mathcal{F}(\alpha) :\simeq \alpha \qquad (\mathcal{F}_8)$$

$$\mathcal{F}(\tilde{\mu}x.c) :\simeq \tilde{\mu}x.\mathcal{F}(c) \qquad (\mathcal{F}_9)$$

$$\mathcal{F}(\pi_i\ s) :\simeq \pi_i\ \mathcal{F}(s) \qquad (\mathcal{F}_{10})$$

$$\mathcal{F}(w \cdot s) :\simeq \mathcal{F}(w) \cdot \mathcal{F}(s) \qquad (\mathcal{F}_{11})$$

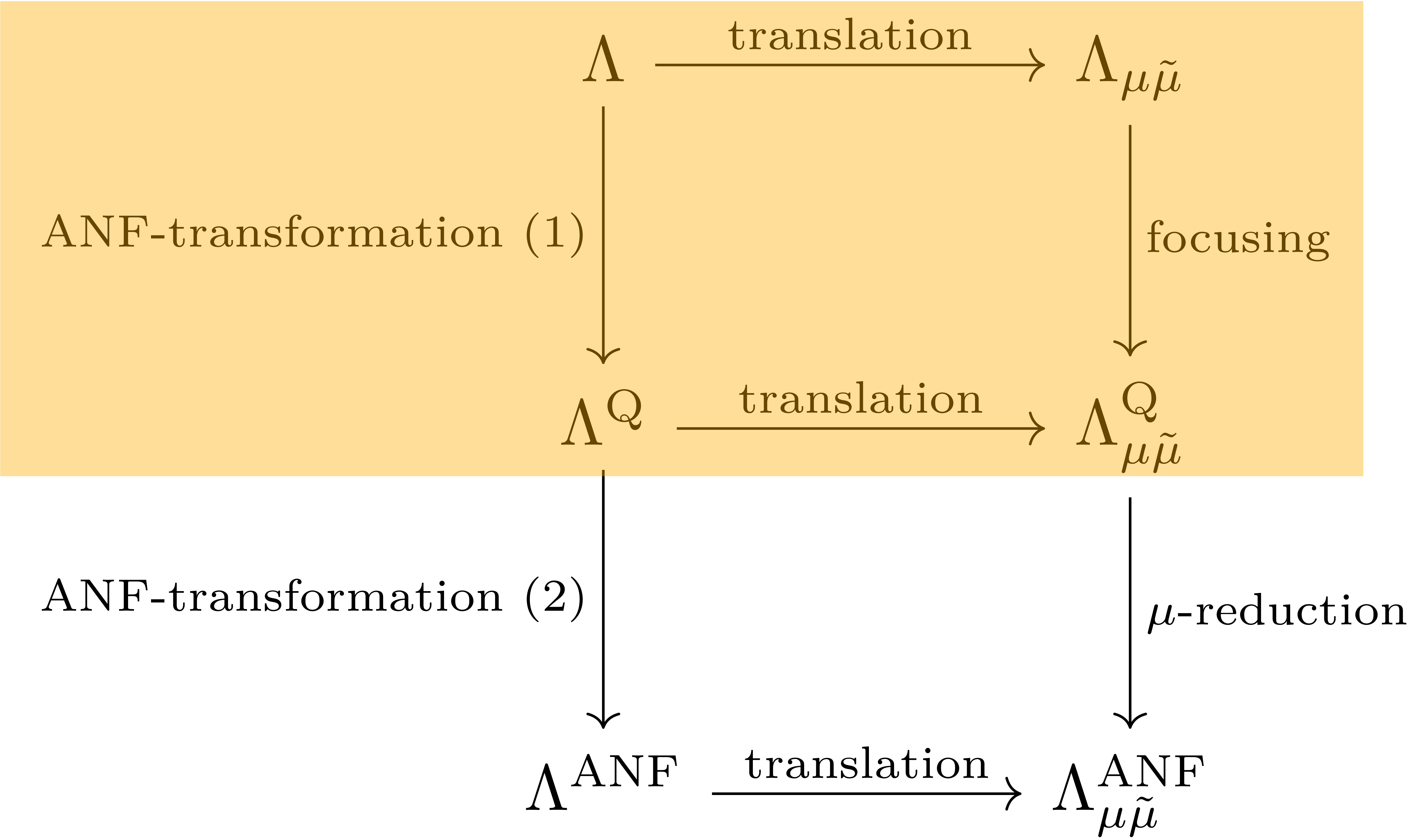
$$\mathcal{F}(t \cdot s) :\simeq \tilde{\mu}x.\langle \mathcal{F}(t) \mid \tilde{\mu}y.\langle x \mid y \cdot \mathcal{F}(s)\rangle\rangle \qquad (\mathcal{F}_{12})$$

Commands

$$\mathcal{F}(\langle t \mid s\rangle) :\simeq \langle \mathcal{F}(t) \mid \mathcal{F}(s)\rangle \qquad (\mathcal{F}_{13})$$

$$\mathcal{F}(\langle t_1 \mid t_2 \cdot s\rangle) :\simeq \langle \mathcal{F}(t_2) \mid \tilde{\mu}x.\langle \mu\alpha.\langle \mathcal{F}(t_1) \mid x \cdot \alpha\rangle \mid \mathcal{F}(s)\rangle\rangle \qquad (\mathcal{F}_{14})$$

Summary



Λ
translation
$\Lambda_{\mu\tilde{\mu}}$
ANF-transformation (1)
focusing
Λ^{Q}
translation
$\Lambda^{Q}_{\mu\tilde{\mu}}$
ANF-transformation (2)
μ-reduction
Λ^{ANF}
translation
$\Lambda^{ANF}_{\mu\tilde{\mu}}$

Future Work

- Extend to call-by-name (easy) and call-by-need/co-need (hard).
- Add control operators to the lambda-calculus to be able to translate in both directions.
- Fully mechanized verification (underway).

Thank you for your attention!